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A Plane-Symmetric Magnetized Inhomogeneous Cosmological Models of Perfect Fluid Distribution with Variable Magnetic Permeability in Lyra Geometry

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Abstract A plane-symmetric magnetized inhomogeneous cosmological model of the universe with time dependent gauge function β for perfect fluid distribution with variable magnetic permeability within the framework of Lyra geometry is investigated. The source of the magnetic field is due to an electric current produced along the z-axis. Thus F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get a deterministic solution of Einstein's modified field equations, the free gravitational field is assumed to be Petrov type-II non-degenerate. For our derived model we obtain the deceleration parameter $q = -1$ as in the case of de Sitter universe. It has been found that the displacement vector $\beta(t)$ behaves like cosmological term Λ in the normal gauge treatment and the solution is consistent with the observations. The displacement vector $\beta(t)$ affects entropy. Some physical and geometric properties of the model are also discussed.

Keywords Cosmology · Variable magnetic permeability · Inhomogeneous universe · Lyra geometry

1 Introduction

The standard Friedman-Robertson-Walker (FRW) cosmological model prescribes a homogeneous and an isotropic distribution for its matter in the description of the present state of the universe. At the present state of evolution, the universe is spherically symmetric and the matter distribution in the universe is on the whole isotropic and homogeneous. But in early stages of evolution, it could have not had such a smoothed picture. Close to the big bang singularity, neither the assumption of spherical symmetry nor that of isotropy can be strictly

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valid. So we consider plane-symmetric, which is less restrictive than spherical symmetry and can provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7, 8], Szafron and Collins [9]. Senovilla [10] obtained a new class of exact solutions of Einstein's equation without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruis and Senovilla [11] have separated out a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable function of r and t as metric coefficients. Dadhich et al. [12] have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple in-homogenization and anisotropization of the former. Recently, Patel et al. [13] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background space-time admits two space-like commuting killing vectors and has separable metric coefficients. Bali and Tyagi [14] obtained a plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electro-magnetic field. Recently, Pradhan et al. [15–19] have investigated plane-symmetric inhomogeneous cosmological models in various contexts.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [20]. Also Harrison [21] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [22]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors [23–32]. Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic [33, 34]. Such fields can be generated at the end of an inflationary epoch [35–39]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [40] had obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Pradhan et al. [41–45] have investigated magnetized viscous fluid cosmological models in various contexts.

Maxwell considered the magnetic permeability ($\bar{\mu}$) to be a constant for a given material. Maxwell considered the spatial gradient of the magnetic field intensity in the steady state to be exclusively determined by a variation in the velocity of the molecular vortices within the magnetic lines of force. To this day, it is assumed that the magnetic permeability is a constant for a given material. But from 'The Double Helix Theory of the Magnetic Field' [46], we must look to a variable magnetic permeability in order to account for variations in magnetic flux density in the steady state, and if we look at the solenoidal magnetic field pattern around a bar magnet, this is not very difficult to visualize. The magnetic field lines are clearly more concentrated at the poles of the magnet than elsewhere. It should be quite obvious that the density of the vortex sea, as denoted by the quantity $\bar{\mu}$, is a variable quantity and that this density visibly varies according to how tightly the magnetic lines of force are packed together [47].

In Einstein's general relativity, the curvature of the space-time is influenced by matter, and it provides the geometrical description of matter. Einstein succeeded in geometrizing gravitation by expressing gravitational potential in terms of metric tensor. In general relativity, spatially homogeneous space-times either belong to Bianchi type or to Kantowaski-Sachs models and interpreted as cosmological models [48]. Partridge and Wilkinson [49] and Ehlers et al. [50] pointed out that spatially homogeneous and isotropic universes can be well described by Friedmann-Robertson-Walker (FRW) model. However, the FRW model has the disadvantage of being unstable near the singularity [51] and it fails to describe the early universe. Therefore spatially homogeneous and anisotropic Bianchi type-I models are undertaken to understand the universe at its early stage of evolution. The idea of geometrizing gravitation by Einstein in 1917 inspired Weyl [52] to develop a theory to geometrize gravitation and electromagnetism. Weyl [52] suggested the first so-called unified field theory based on a generalization of Riemannian geometry. With its backdrop, it would seem more appropriate to call Weyl's theory a geometrized theory of gravitation and electromagnetism, instead a unified field theory. It is not clear as to what extent the two fields have been unified, even though they acquire (different) geometrical significance in the same geometry. The theory was never taken seriously in as much as it was based on the concept of non-integrability of length transfer; and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl's geometry provides an interesting example of non-Riemannian connections, and Folland [53] has given a global formulation of Weyl manifolds clarifying considerably many of Weyl's basic ideas thereby.

In 1951, Lyra [54] proposed a modification of Riemannian geometry by introducing a gauge function into the structure-less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. But in Lyra's geometry, unlike that of Weyl, the connection is metric preserving as in Riemannian; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the "normal" gauge the curvature scalar is identical to that of Weyl. In consecutive investigations Sen and co-worker [55, 56] proposed a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry. It is, thus, possible to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory, however, without the inconvenience of non-integrability length transfer. Halford [57] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [58] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits as the Einstein's general theory of relativity. Several authors Sen and Vanstone [59], Bhamra [60], Karade and Borikar [61], Kalyanshetti and Wagmode [62], Reddy and Innaiah [63], Beesham [64], Reddy and Venkateswarlu [65], Soleng [66], studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one for convenience and there is no *a priori* reason for it. Beesham [67] considered Friedmann-Robertson-Walker (FRW) models with time dependent displacement field. Singh and co-workers [68–72] studied Bianchi-type I, III, Kantowaski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [66] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [73–75] or contain a special vacuum field, which together with the gauge vector term, may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.

Recently, Pradhan et al. [76–81], Casama et al. [82], Rahaman et al. [83, 84], Bali and Chandnani [85], Kumar and Singh [86], Singh [87], Rao, Vinutha and Santhi [88] and Pradhan [89] have studied cosmological models based on Lyra's geometry in various contexts. Recently Bali [90] has obtained Bianchi type V magnetized string dust universe with variable magnetic permeability. With these motivations, in this paper, we have obtained exact solutions of Einstein's modified field equations for perfect fluid distribution with variable magnetic permeability in plane symmetric inhomogeneous space-time within the frame work of Lyra's geometry for time varying displacement vector $\beta(t)$. This paper is organized as follows. In Sect. 1 the motivation for the present work is discussed. The metric and the field equations are presented in Sect. 2. Section 3 deals with the solutions of field equations. Some physical and geometric properties of the model are described in Sect. 4. Finally, discussion and concluding remarks are given in Sect. 5.

2 The Metric and Field Equations

We consider the metric in the form of Marder [91]

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \tag{1}$$

where the metric potential A, B and C are functions of x and t. The energy momentum tensor is taken as

$$T_i^j = (\rho + p)v_i v^j + pg_i^j + E_i^j, \tag{2}$$

where E_i^j is the electro-magnetic field given by Lichnerowicz [92] as

$$E_i^j = \bar{\mu}\left[h_l h^l\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right]. \tag{3}$$

Here ρ and p are the energy density and isotropic pressure respectively and v^i is the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1. \tag{4}$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}{}^*F_{ji}v^j, \tag{5}$$

where ${}^*F_{ij}$ is the dual electro-magnetic field tensor defined by Synge [93]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{6}$$

F_{ij} is the electro-magnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density. The coordinates are considered to be comoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = \frac{1}{A}$. We consider that the current is flowing along the z-axis so that $h_3 \neq 0$, $h_1 = 0 = h_2 = h_4$. The only non-vanishing component of F_{ij} is F_{12}. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{7}$$

and

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = 0 \tag{8}$$

require that F_{12} be function of x alone. We assume that the magnetic permeability as a function of x and t both. Here the semicolon represents a covariant differentiation.

The field equations, in normal gauge for Lyra's manifold, obtained by Sen [55] as

$$R_i^j - \frac{1}{2} g_i^j R + \frac{3}{2} \phi_i \phi^j - \frac{3}{4} g_i^j \phi_k \phi^k = -8\pi T_i^j, \tag{9}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta(t)), \tag{10}$$

where other symbols have their usual meaning as in Riemannian geometry.

For the line element (1), the field equation (9) with (2) and (10) lead to the following system of equations

$$\begin{aligned} 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right) = \frac{1}{A^2}\Bigg[&-\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) \\ &+ \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{B_1 C_1}{BC} - \frac{B_4 C_4}{BC}\Bigg] - \frac{3}{4}\beta^2, \end{aligned} \tag{11}$$

$$8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right) = \frac{1}{A^2}\left[-\left(\frac{A_4}{A}\right)_4 + \left(\frac{A_1}{A}\right)_1 - \frac{C_{44}}{C} + \frac{C_{11}}{C}\right] - \frac{3}{4}\beta^2, \tag{12}$$

$$8\pi\left(p - \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right) = \frac{1}{A^2}\left[-\left(\frac{A_4}{A}\right)_4 + \left(\frac{A_1}{A}\right)_1 - \frac{B_{44}}{B} + \frac{B_{11}}{B}\right] - \frac{3}{4}\beta^2, \tag{13}$$

$$\begin{aligned} 8\pi\left(\rho + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right) = \frac{1}{A^2}\Bigg[&-\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) \\ &+ \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC}\Bigg] + \frac{3}{4}\beta^2, \end{aligned} \tag{14}$$

$$0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right), \tag{15}$$

where the sub indices 1 and 4 in A, B, C and elsewhere indicate ordinary differentiation with respect to x and t, respectively.

The energy conservation equation $T^i_{i;j} = 0$ leads to

$$\rho_4 + (\rho + p)\left(\frac{2A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{16}$$

and conservation of R.H.S. of (9) leads to

$$\left(R_i^j - \frac{1}{2}g_i^j R\right)_{;j} + \frac{3}{2}(\phi_i\phi^j)_{;j} - \frac{3}{4}(g_i^j\phi_k\phi^k)_{;j} = 0. \tag{17}$$

Equation (17) reduces to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j} + \phi^l\Gamma_{lj}^j\right] + \frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j} - \phi_l\Gamma_{ij}^l\right] - \frac{3}{4}g_i^j\phi_k\left[\frac{\partial\phi^k}{\partial x^j} + \phi^l\Gamma_{lj}^k\right]$$
$$-\frac{3}{4}g_i^j\phi^k\left[\frac{\partial\phi_k}{\partial x^j} - \phi_l\Gamma_{kj}^l\right] = 0. \tag{18}$$

Equation (18) is identically satisfied for $i = 1, 2, 3$. For $i = 4$, (18) reduces to

$$\frac{3}{2}\beta\left[\frac{\partial(g^{44}\phi_4)}{\partial x^4} + \phi^4\Gamma_{44}^4\right] + \frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial t} - \phi_4\Gamma_{44}^4\right] - \frac{3}{4}g_4^4\phi_4\left[\frac{\partial\phi^4}{\partial x^4} + \phi^4\Gamma_{44}^4\right]$$
$$-\frac{3}{4}g_4^4 g^{44}\phi^4\left[\frac{\partial\phi_4}{\partial t} - \phi^4\Gamma_{44}^4\right] = 0, \tag{19}$$

which leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(\frac{2A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{20}$$

3 Solution of the Field Equations

Equations (11)–(13) lead to

$$\left(\frac{A_4}{A}\right)_4 - \frac{B_{44}}{B} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_4C_4}{BC}$$
$$= \left(\frac{A_1}{A}\right)_1 + \frac{C_{11}}{C} - \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_1C_1}{BC} = k \text{ (constant)} \tag{21}$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu}B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{C_{44}}{C}. \tag{22}$$

Equations (11)–(15) represent a system of five independent equations in seven unknowns A, B, C, ρ, p, F_{12} and $\beta(t)$. For the complete determination of these unknowns two more conditions are needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit. In the present case, we assume that the metric is Petrov type-II non-degenerate.

This requires that

$$\left(\frac{B_{11}+B_{44}+2B_{14}}{B}\right)-\left(\frac{C_{11}+C_{4}+2C_{14}}{C}\right)$$
$$=\frac{2(A_1+A_4)(B_1+B_4)}{AB}-\frac{2(A_1+A_4)(C_1+C_4)}{AC}. \tag{23}$$

Let us consider that

$$\begin{aligned} A &= f(x)\lambda(t),\\ B &= g(x)\mu(t),\\ C &= g(x)\nu(t). \end{aligned} \tag{24}$$

Using (24) in (15) and (23), we get

$$\left[\frac{\frac{g_4}{g}-\frac{f_1}{f}}{\frac{g_1}{g}}\right]=\left[\frac{\frac{2\lambda_4}{\lambda}}{\frac{\mu_4}{\mu}+\frac{\nu_4}{\nu}}\right]=b \text{ (constant)} \tag{25}$$

and

$$\frac{\frac{\mu_{44}}{\mu}-\frac{\nu_{44}}{\nu}}{\frac{\mu_4}{\mu}-\frac{\nu_4}{\nu}}-\frac{2\lambda_4}{\lambda}=2\left(\frac{f_1}{f}-\frac{g_1}{g}\right)=L \text{ (constant)}. \tag{26}$$

Equation (25) leads to

$$f=ng^{(1-b)} \tag{27}$$

and

$$\lambda=m(\mu\nu)^{\frac{b}{2}}, \tag{28}$$

where m and n are constants of integration. Equations (21), (24) and (26) lead to

$$\left(\frac{b}{2}-1\right)\frac{\mu_{44}}{\mu}+(b-1)\frac{\mu_4\nu_4}{\mu\nu}=k, \tag{29}$$

and

$$(2-b)\frac{g_{11}}{g}+(3b-4)\frac{g_1^2}{g^2}=k. \tag{30}$$

Let us assume

$$\mu=e^{U+V}, \tag{31}$$

and

$$\nu=e^{U-V}. \tag{32}$$

Equations (26), (31) and (32) lead to

$$V_4=Me^{Lt+2(b-1)U}, \tag{33}$$

where M is constant. From equations (29), (31), (32) and (33), we have

$$(b-1)U_{44}+2(b-1)U_4^2-2bMe^{Lt+2(b-1)U}U_4-MLe^{Lt+2(b-1)U}=k. \tag{34}$$

If we put $e^{2U} = \xi$ in (34), we obtain

$$\frac{(b-1)}{2}\frac{d^2\xi}{dt^2} - M\frac{d}{dt}(e^{Lt}\xi^b) = k\xi. \tag{35}$$

If we consider $\xi = e^{qt}$, then (35) leads to

$$\frac{(b-1)}{2}g^2 e^{qt} - M\frac{d}{dt}(e^{Lt}e^{qbt}) = ke^{qt}, \tag{36}$$

which again reduces to

$$q = \frac{L}{1-b}, \tag{37}$$

and

$$k = \frac{L(L+2M)}{2(b-1)}. \tag{38}$$

Thus

$$U = \frac{Lt}{2(1-b)}. \tag{39}$$

Equations (33) and (39) reduce to

$$V = Mt + \log N, \tag{40}$$

where N is an integrating constant. Equation (29) leads to

$$g = \beta \cosh^{\frac{2-b}{2(b-1)}}(\alpha x + \delta), \tag{41}$$

where

$$\alpha = \frac{\sqrt{2(3b-4)(1-b)}}{(2-b)}, \tag{42}$$

and β, δ being constants of integration. Hence

$$f = n\beta \cosh^{\frac{b-2}{2(b-1)}}(\alpha x + \delta), \tag{43}$$

$$\lambda = me^{\frac{Lbt}{2(1-b)}}, \tag{44}$$

$$\mu = e^{\frac{Lbt}{2(1-b)} + Mt + \log N}, \tag{45}$$

$$\nu = e^{\frac{Lbt}{2(1-b)} - Mt - \log N}. \tag{46}$$

Therefore, we have

$$A = f\lambda = mn\beta e^{\frac{Lbt}{2(1-b)}} \cosh^{\frac{b-2}{2}}(\alpha x + \delta), \tag{47}$$

$$B = g\mu = N\beta e^{\left(\frac{L}{1-b}+2M\right)\frac{t}{2}} \cosh^{\frac{2-b}{2(b-1)}}(\alpha x + \delta), \tag{48}$$

$$C = g\nu = \frac{\beta}{N}e^{\left(\frac{L}{1-b}-2M\right)\frac{t}{2}} \cosh^{\frac{2-b}{2(b-1)}}(\alpha x + \delta). \tag{49}$$

By using the transformation

$$\begin{aligned} X &= x + \frac{\delta}{\alpha}, \\ Y &= y, \\ Z &= z, \\ T &= t, \end{aligned} \tag{50}$$

the metric (1) reduces to the form

$$\begin{aligned} ds^2 = K^2 \cosh^{b-2}(\alpha X) e^{\frac{LTb}{1-b}} (dX^2 - dT^2) \\ + G^2 \cosh^{\frac{2-b}{b-1}}(\alpha X) e^{\left(\frac{L}{1-b}+2M\right)T} dY^2 \\ + H^2 \cosh^{\frac{2-b}{b-1}}(\alpha X) e^{\left(\frac{L}{1-b}-2M\right)T} dZ^2, \end{aligned} \tag{51}$$

where $K = mn\beta$, $G = N\beta$ and $H = \frac{\beta}{N}$.

Since the magnetic permeability is a variable quantity, we have assumed it as

$$\bar{\mu} = e^{-\left(\frac{L}{1-b}+2M\right)T}. \tag{52}$$

Thus $\bar{\mu} \to 0$ as $T \to \infty$ and $\bar{\mu} = 1$ when $T \to 0$. Zel'dovich [94] has explained that $\rho_s/\rho_c \sim 2.5 \times 10^{-3}$, where ρ_s is the mass density and ρ_c the critical density then the bodies frozen in plasma would change their density like a^{-2} i.e. like t^{-1} in the radiation dominated universe where a is the radius of the universe.

4 Some Physical and Geometric Properties

The physical parameters, pressure (p) and density (ρ), for the model (51) are given by

$$\begin{aligned} 8\pi p = \frac{1}{K^2} e^{\frac{mnbT}{b-1}} \cosh^{2-b}(\alpha X) \left[\frac{(2-b)^2\alpha^2}{4(b-1)} \left\{ 1 + \frac{2-b}{b-1} \tanh^2(\alpha X) \right\} \right. \\ \left. - \frac{L^2}{4(1-b)^2} - M^2 \right] - \frac{3}{4}\beta^2, \end{aligned} \tag{53}$$

$$\begin{aligned} 8\pi \rho = \frac{1}{K^2} e^{\frac{mnbT}{b-1}} \cosh^{2-b}(\alpha X) \left[\frac{(2-b)\alpha^2}{2(b-1)} \left\{ \frac{b}{b-1} \tanh^2(\alpha X) - 1 \right\} \right. \\ \left. + \frac{L^2(2b-1)}{4(1-b)^2} - M^2 - \frac{ML}{(1-b)} \right] + \frac{3}{4}\beta^2. \end{aligned} \tag{54}$$

From (20) we have

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2 \left(\frac{2A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0,$$

which gives either $\beta = 0$ or $\beta_4 + \beta(\frac{2A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}) = 0$.

Therefore

$$\frac{\beta_4}{\beta} = -\left(\frac{2A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{55}$$

which reduces to

$$\frac{\beta_4}{\beta} = \frac{(1+b)L}{2(b-1)} T^2. \tag{56}$$

Integrating (56), we obtain

$$\beta = e^{\frac{(1+b)L}{6(b-1)} T^3}. \tag{57}$$

The non-vanishing component F_{12} of the electromagnetic field tensor is given by

$$F_{12} = \sqrt{\frac{ML}{4\pi(1-b)}} G \cosh^{\frac{2-b}{2(b-1)}}(\alpha X), \tag{58}$$

which is function of x alone. So it is consistent as the Maxwell's equations (7) and (8) require F_{12} to be function of x alone.

The scalar of expansion (θ) calculated for the flow vector (v^i) is given by

$$\theta = \frac{L(b+2)}{2K(1-b)} e^{\frac{LbT}{2(b-1)}} \cosh^{\frac{(2-b)}{2}}(\alpha X). \tag{59}$$

The shear scalar (σ^2), acceleration vector $(\dot{v}_i)$, deceleration parameter q and proper volume (V^3) for the model (51) are given by

$$\sigma^2 = \frac{(L^2 + 12M^2)}{12K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X), \tag{60}$$

$$\dot{v}_i = \left(\frac{\alpha(b-2)}{2}\tanh(\alpha X), 0, 0, 0\right), \tag{61}$$

$$q = -\frac{V_{44}/V}{V_4^2/V^2} = -1, \tag{62}$$

$$V^3 = \sqrt{-g} = K^2 G H e^{\frac{L(b+1)T}{(1-b)}} \cosh^{\frac{(b-2)(b-3)}{(b-1)}}(\alpha X). \tag{63}$$

From (59) and (60), we have

$$\frac{\sigma^2}{\theta^2} = \frac{(L^2 + 12M^2)(1-b^2)}{3L^2(b+2)^2} = \text{constant}. \tag{64}$$

The rotation ω is identically zero and the non-vanishing component of conformal curvature tensor are given by

$$C_{(1212)} = \frac{1}{6K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X)\left[b\alpha - \frac{L^2}{4b} + 3ML - 2M^2\right], \tag{65}$$

$$C_{(1313)} = \frac{1}{6K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X)\left[b\alpha - \frac{L^2}{4b} - 3ML - 2M^2\right], \tag{66}$$

$$C_{(2323)} = \frac{1}{3K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X)\left[-b\alpha + \frac{L^2}{4b} + \frac{ML}{(1-b)} + 2M^2\right], \tag{67}$$

$$C_{(1224)} = \frac{ML}{2K^2} e^{\frac{LbT}{(b-1)}} \cosh^{(2-b)}(\alpha X). \tag{68}$$

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{A_4}{A} = \frac{Lb}{2(1-b)}, \tag{69}$$

$$H_y = \frac{B_4}{B} = \frac{1}{2}\left(\frac{L}{1-b} + 2M\right), \tag{70}$$

$$H_z = \frac{C_4}{C} = \frac{1}{2}\left(\frac{L}{1-b} - 2M\right). \tag{71}$$

The dominant energy condition is given by Hawking and Ellis [95]

(i) $\rho - p \geq 0$
(ii) $\rho + p \geq 0$

lead to

$$e^{\frac{mnbT}{(b-1)}}\left[\frac{(2-b)\alpha^2}{(b-1)}\left\{\frac{(3b-2)}{4(1-b)}\tanh^2(\alpha X) + \frac{b-4}{2}\right\}\right.$$
$$\left. + \frac{L^2 b}{(1-b)^2} - \frac{ML}{(1-b)}\right] + 3K^2\beta^2\cosh^{(b-2)}(\alpha X) \geq 0 \tag{72}$$

and

$$\frac{(2-b)\alpha^2}{4(b-1)^2}\left[(b^2 - 2b + 4)\tanh^2(\alpha X) + b(1-b)\right]$$
$$\geq \frac{L^2}{2(1-b)} + 2M^2 + \frac{ML}{(1-b)}, \tag{73}$$

respectively.

5 Discussion and Concluding Remarks

We have obtained a new plane-symmetric inhomogeneous cosmological model of electromagnetic perfect fluid as the source of matter with variable magnetic permeability. Generally the model represents expanding, shearing, non-rotating and Petrov type-II non-degenerate universe in which the flow vector is geodesic. We find that the model starts expanding at $T = 0$ and goes on expanding indefinitely. However, if $b < 0$ the process of contraction starts and at $T = \infty$ the expansion stops. For large values of T, the model is conformally flat and Petrov type-II non-degenerate otherwise. Since $\frac{\sigma}{\theta} =$ constant, hence the model does not approach isotropy. The electromagnetic field tensor does not vanish when $L \neq 0$, $M \neq 0$, and $G \neq 0$.

In spite of homogeneity at large scale our universe is inhomogeneous at small scale, so physical quantities being position-dependent are more natural in our observable universe if we do not go to super high scale. Our derived model shows this kind of physical importance. From (62), it follows that our model of the universe is consistent with recent observations which reveal that the present universe is in accelerating phase and the deceleration parameter lies somewhere in the range $-1 < q \le 0$. For our model $q = -1$ as in the case of de Sitter universe.

It is possible to discuss entropy in our universe. In thermodynamics the expression for entropy is given by

$$TdS = d(\rho V^3) + p(dV^3), \tag{74}$$

where $V^3 = A^2BC$ is the proper volume in our case. To solve the entropy problem of the standard model, it is necessary to treat $dS > 0$ for at least a part of evolution of the universe.

Hence (74) reduces to

$$TdS = \rho_4 + (\rho + p)\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) > 0. \tag{75}$$

The conservation equation $T^j_{i;j} = 0$ for (1) leads to

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{76}$$

Therefore, (75) and (76) lead to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\left(2\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) < 0, \tag{77}$$

which gives to $\beta < 0$. Thus, the displacement vector $\beta(t)$ affects entropy because for entropy $dS > 0$ leads to $\beta(t) < 0$.

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